Analytical versus Monte Carlo Description of Cherenkov Contribution in Air Showers

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Abstract

The CORSIKA simulation code is used to calculate the longitudinal profile of Cherenkov photons for showers at the highest energies. The results are compared to analytical calculations based on the longitudinal shower size profile and electron energy distribution. A new, universal parametrization of the electron energy distribution in high-energy showers is presented. This parametrization allows us to derive the longitudinal Cherenkov profile both in a purely analytical way and *a posteriori* using the longitudinal particle number provided by CORSIKA. Necessary corrections in the normalization due to the specific energy threshold used in the simulation are discussed. The parametrization can be used in calculations e.g. for fluorescence telescope observations and shower reconstruction.

1. Introduction

For the determination of the primary energy using the fluorescence observation technique, a good knowledge of the Cherenkov background in the measured signal is mandatory. The CORSIKA code [3] has been adapted to calculate the longitudinal Cherenkov profile while generating the shower. Alternatively, the number of Cherenkov photons dN_γ produced per slant depth dX in a shower at depth X can be calculated analytically by

$$\frac{dN_\gamma}{dX}(X) = \int_{\ln E_t}^{\infty} N(X)\ y_\gamma(h,E)\ f(X,E)\ d\ln E \tag{1}$$

$N(X)$ is the charged particle number as function of depth X, which will be taken from CORSIKA. $y_\gamma(h,E)$ denotes the Cherenkov yield of a single particle with energy E at altitude h in the atmosphere and E_t the local Cherenkov energy threshold, which depends on the refractive index $\eta = \eta(h)$ of air. For a given shower geometry, $h = h(X)$ follows from the atmospheric model assumed (US standard atmosphere in the following). y_γ shows the well-known sharp threshold

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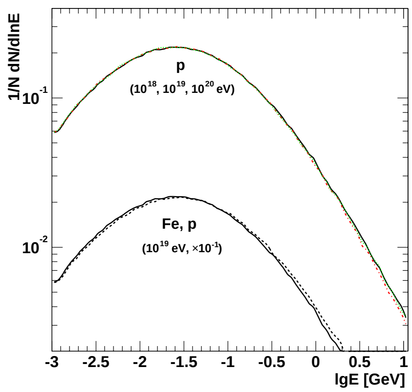

Fig. 1. Electron energy spectra at $s = 1$. Simulations with CORSIKA for different primary energies and primary particles.

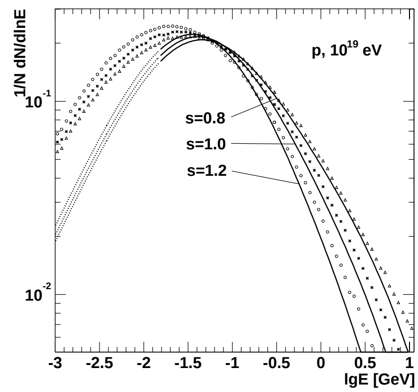

Fig. 2. Electron energy spectra from CORSIKA (symbols) for $s = 0.8$, 1.0, and 1.2 compared to parametrizations according to Hillas [4] (lines).

dependence. $f(X, E) = dN/(N \ d\ln E)$ is the (normalized) differential energy spectrum at shower stage X, which can be obtained e.g. by shower calculations [5]. A parametrization of the electron energy spectrum, depending only on the shower age $s = 3X/(X+2X_{max})$, was provided by Hillas based on 100 GeV photon shower simulations using a low-energy particle cutoff of 50 keV [4]. Traditionally this approximation is used to calculate the Cherenkov contamination of fluorescence light signals from high-energy showers, see for example [1].

The plan of the paper is as follows. At first, energy spectra obtained from CORSIKA are studied and compared to those given in [4]. A new parametrization, better reproducing the CORSIKA spectra, is introduced. The resulting predictions of the longitudinal Cherenkov profile using the different spectrum parametrizations are then compared to a full CORSIKA simulation.

2. Electron Energy Spectra

In Fig. 1, electron energy spectra at shower maximum obtained by CORSIKA are shown for different combinations of primary energy and mass. The normalization of the energy spectra, being important for the final calculation of the Cherenkov profile, is discussed below. The spectral shape does not depend in the considered energy range on primary parameters, which allows a parametrization valid for a large range of primary energies and masses. A dependence of the spectra on the shower age has already been considered in the parametrizations given by Hillas [4]. In Fig. 2, these spectra are compared to the CORSIKA results for different shower ages. Given the fact that the parametrizations were obtained for low-energy primary photons, a larger disagreement to CORSIKA above ener-

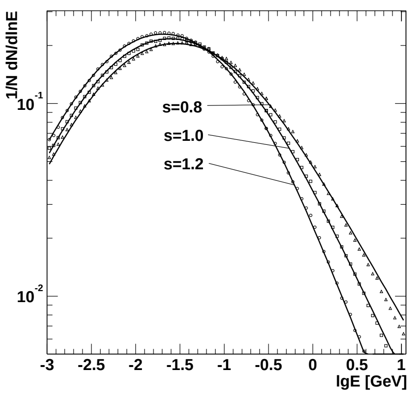

Fig. 3. CORSIKA energy spectra (see also Fig. 2) compared to the new parametrization.

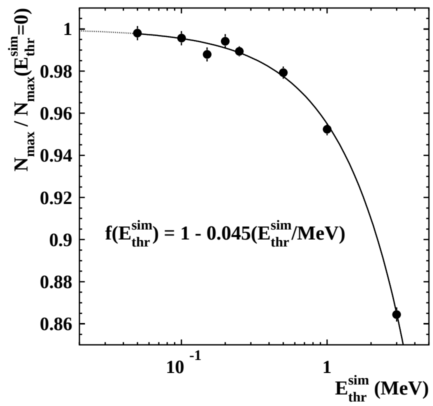

Fig. 4. Shower particle content as function of simulation energy threshold (see text) and its parametrization.

gies of 15 MeV (the lower validity limit given in [4]) might have been expected. A better description of the CORSIKA energy spectra, taking the age dependence into account and being also applicable to lower energies, has been achieved using the functional form

$$f_{para}(s, E) = a_0 \cdot \frac{E}{(E + a_1)(E + a_2)^s} \tag{2}$$

As can be seen in Fig. 3, the CORSIKA spectra can be reproduced well using $a_1 = 6.879 - 2.092 \cdot s$ and $a_2 = 122.0$ (for E in MeV). Another independently developed functional form is discussed in [2].

The energy spectra shown in Fig. 3 have been normalized according to

$$f_{para}(s, E) = \frac{1}{N}\frac{dN}{d\ln E} \quad , \quad \text{with} \quad \int_{\ln E_{thr}^{sim}}^{\infty} f_{para}(s, E) d\ln E = 1, \tag{3}$$

where E_{thr}^{sim} is the energy threshold adopted in the simulation (1 MeV in the examples shown). This normalization is necessary to be consistent with the shower size profile, as N provided by the simulation refers only to the particles above this energy threshold. As an example, the reduction of the maximum particle number with increasing energy threshold is illustrated in Fig. 4. Hillas' parametrization does not give an adequate description of the particle spectrum at energies below 20 MeV. Therefore it is not suited for calculations based on Eq. (1) if a threshold different from 50 keV was used for calculating the longitudinal shower profile.

3. Longitudinal Cherenkov Profile

The longitudinal Cherenkov profile generated by CORSIKA for an exemplary primary proton of 10^{19} eV is shown in Fig. 5 together with the results based

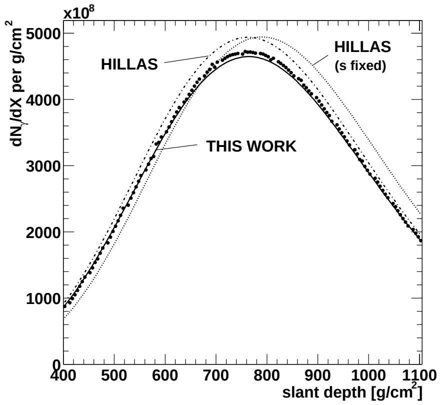

Fig. 5. Longitudinal Cherenkov profile obtained by CORSIKA, different parametrizations (see text).

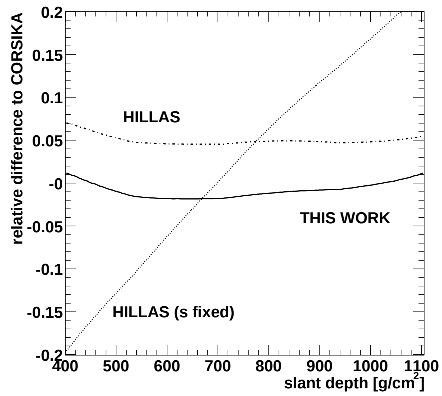

Fig. 6. Relative difference of the profiles shown in Fig. 5 to the CORSIKA profile.

on the different parametrizations. The relative difference of the profiles to the CORSIKA one is displayed in Fig. 6. The calculation labeled "Hillas (s fixed)" employs Hillas' parametrization for $s = 1$ only, as often used (see e.g. [1]). This approximation leads to a shift of the Cherenkov profile by about $30-40$ g/cm^2 towards larger depths, due mainly to the neglected reduction of high-energy electrons with growing age. It could be cured to a large extend by taking the s-dependence of [4] into account. However, the predicted Cherenkov production exceeds the CORSIKA one by $\simeq 5\%$. The best agreement is obtained using the new parametrization. Around the Cherenkov profile maximum, the deficit is less than $1-2\%$. The angular dispersion of particles, effectively increasing the Cherenkov yield per traversed depth dX along the axis, has not yet been taken into account in the analytical approaches. This might result in a modest increase of the predicted curves.

4. Conclusions

CORSIKA can be used to directly calculate the Cherenkov longitudinal profile. For analytical applications, an improved parametrization of the electron energy spectra is proposed based on CORSIKA simulations. It can also be used to infer the Cherenkov profile from a given charged particle output.

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